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LECTURE FIVE as delivered in Aberdeen

Order.

1. The analysis of the exact sciences that I have given so far started off from a definition of classical physics as a formalism which, even though it could be strictly falsified by certain conceivable events, could be falsified by actual events only under certain conditions acknowledged by the personal judgment of the scientist. Another element of personal knowledge appeared to be claimed by the scientist when appraising the internal rational qualities of a theory. The joint assessment of the theory's correspondence to experience together with its intellectual attractiveness was the ground on which the ultimate acknowledgement of its credibility was seen to rest.

As we went on to quantum mechanics we noted that falsifiability of theory by some conceivable event was lost and that the assessment of correspondence between theory and actual events had become massively and ostensibly dependent on personal judgment. All quantitative statements of probability were recognised as exact scientific laws which prescribed a personal expectation of and subsequent reaction to events without making any unambiguous predictions about them.

But there are other areas in the exact sciences where personal appraisal plays a prominent part - and where, consequently, as in quantum mechanics, the test of falsifiability is wanting. These consist of mathematical formalisms in which certain variable magnitudes are said to be small integers but are otherwise left indeterminate. Representatives of this type of formalism are the law of simple proportions in chemistry and the law of rational indices in crystallography.

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The law of simple chemical proportions asserts that if there are a variety of compounds into which the chemical elements S, T, U ... combine, it is possible to discover for each element a characteristic quantity called the atomic weight, the use of

which as the measure for the quantity of the element contained in any compound will result in the expression of the elementary composition of the compound by a set of small integers.

There are two inter-related terms by which the reference of such a formalism to experience relies more heavily on acts of personal appraisal than does the formalism of classical physics. They are the terms 'small' and 'integer'. To ascertain a simple proportion of integers, from a measurement of weight or length demands that we go beyond the method of ascertaining measured quantities from sets of instrument readings, as we do for the verification of predictions in classical dynamics. We must take the further step of identifying arithmetic proportions of measured quantities with integer fractions. While the transition from sets of instrument readings to numbers accepted as measured, can be formalised up to a point by the assumption of random errors to account for the spread of instrument readings, there is no formal rule for obtaining the integer fractions to be accepted as corresponding to any particular proportion of measured numbers.

This act is complicated, moreover, by the inevitable demand that the integer be small. It may be considered obvious that if the relation of the measured proportions of carbon to hydrogen in a sample of methane and a sample of ethylene comes out at 0.504 with a probable error of 0.04, we should assume that it is to be represented by the integer fraction 1/2; but this is only so because we implicitly assume that the ratio must be simple, i.e. made up of <u>small</u> integers. Much closer approximations would be offered by taking the ratios of larger integers. By choosing from these we could always achieve a perfect fit, as we would by taking 1008 : 2000 to represent the measured ratio 0.504.

It is meaningless therefore to speak of establishing a correspondence between measured quantities and integers unless the condition is included that the integers should be small or their fraction simple. A formalism containing unknown quantities which stand for empirically established <u>simple integer fractions</u> relies on our capacity for identifying such fractions from measured numbers in which they are reflected. In accepting as significant a law of nature such as that of simple chemical proportions we claim that we can evaluate observed magnitudes in terms of simple integer fractions. The knowledge to which we lay claim by asserting such laws contains at this point an additional element of our personal participation. The law of rational indices in crystallography says that we can find for each crystal a system of three (not always rectangular) co-ordinates so that it will be possible further to choose a characteristic length as a unit for measuring the intercepts of crystal planes on these co-ordinates with the result that all intercepts of crystal planes will be found to be small integers.

The additional gap opened up in the correspondence between theory and experience by the representation of experience in terms of integers can be illustrated best by an example in which a proposed representation of this kind is still under controversial discussion. Eddington deduced that the "fine structure constant" (which in the usual symbols has the formula $\frac{hc}{2\pi e^2}$) is equal to the integer 137; but no degree of approximation of the measured number to this theoretical value will convince those who reject the theory that the number is an integer.

To the extent to which the attribute of simplicity is vague, the demands which a law of simple proportions makes on

experience are indeterminate. If future observations of chemical proportions could be represented only by larger integers than those found apposite to hitherto analysed compounds we may feel increasingly disappointed in the theory and be eventually altogether discouraged from relying on it. But the process would resemble more the gradual relinquishing of a supposed statistical law which has repeatedly failed to find corroboration, than the rejection of an unambiguous theory which has met with a series of conflicting observations.

4. It is true that the chemical analysis of a substance with a high molecular weight may lead to proportions described by large integers. The end-group of a long chain of carbon atoms may be formed by some element X, so that the proportion of X to carbon (measured in atomic weight units) may be 1:1000 or even higher. When a chemical analysis is interpreted in these terms we no longer rely on the law of simple proportions but on the atomic theory which has come to replace it as the conceptual framework of chemistry. Atoms can be counted, and their counting would necessarily lead to integer proportions of chemical compounds. Proportions obtained by counting are observed integer fractions which need not be simple. Indeed, if we could count the number of sodium and chlorine particles in a crystal of rock salt we would find a slight excess of one or the other kind of particles and the proportion of the two would be something like 1,000,000,000:1,000,000,001. Therefore, when we meet with chemical proportions which cannot be expressed by small integers we may yet interpret them as integer proportions if this appears justified by more direct evidence concerning the atomic structure of the analysed substances. Similarly, the law of rational crystallographic indices is today no longer refutable by measurements regarding the position of crystal planes. If we obtain, as we sometimes do, intercepts

the ratios of which cannot be represented by simple integer fractions, we explain the result in terms of the atomic patterns on which the theory of crystals today ultimately relies.

5. But we must remember that the laws of simple proportions were established both in chemistry and crystallography before the atomic theory was invoked for their explanation, or at any rate, before it was independently established, so that their subsequent confirmation and reinterpretation by the Xray analysis of atomic arrangements shows that the holding of scientific knowledge in the form of such laws was justified. The personal judgment involved in evaluating measured proportions as simple integer ratios may be therefore regarded as forming part of the exact sciences. As a scientist I must credit myself with the capacity of establishing a personal knowledge of definite simple integer ratios, corresponding to the proportions of given measured magnitudes.

6. In addition to assessments of probability, and the formulation of small integer proportions, a third form of personal knowledge to which scientists lay claim, will be illustrated by the elaboration of the mathematical concept of symmetry in crystallographic theory. This theory has developed today into a special branch of geometry but this ultimate framework was arrived at through a series of earlier stages which were originally founded on quite primitive appreciations of regularity. From earliest times men were fascinated by stones of distinctive shapes. Regularity is one of the distinctive characteristics which pleases the eye and stimulates the imagination. Stones, bounded on many sides by plane surfaces which met in straight edges, attracted attention, particularly if they were also beautifully coloured like rubies, sapphires or emeralds. This first attraction held the

intimation of a still hidden and greater significance, which the primitive mind expressed by ascribing magical powers to gems. Later, it stimulated the scientific study of crystals, which by the close of the 17th century had established and elaborated in formal terms the following four appraisals that are inherent in any intelligent appreciation of crystals. (1) That crystals have a characteristic shapeliness which distinguishes them from other objects. (2) That there are certain kinds of crystals, each kind being represented by individual specimens and (3) That there is some characteristic principle connecting the regularities observed within one crystal specimen and within all the specimens of one kind of crystals. (4) That the different kinds of crystals show different degrees of regularity.

The formalism suggested by the founders of modern crystallography applied to all four points simultaneously. It sets up an ideal of shapeliness, by which it classifies solid bodies into such as tend to fulfil this ideal and others in which no such shapeliness is apparent. The first are crystals, the second the shapeless (or amorphous) non-crystals, like glass. Each individual crystal is taken to represent an ideal of regularity, all actual deviations from which are regarded as irrelevant. This ideal shape is found by assuming that approximately plane surfaces of crystals are geometrical planes which extend to the straight edges in which such planes must meet, thus bounding the crystal from all sides. This formalisation defines a polyhedron which is taken to be the theoretical shape of a crystal specimen. It embodies only such aspects of the specimen as are deemed regular and in respect to these it is required to fit the facts of experience; but otherwise however widely the crystal specimen deviates from the theory, this will be put down as a shortcoming of the crystal and not of the theory.

To each crystal specimen there is thus assigned a different

ideal polyhedron and crystallographic theory proceeds next to discover a principle characterising the regularity of these polyhedra. This principle is found residing in the symmetry of crystals. 'Symmetry' is a word with connotations almost as wide as 'order'. When applied to objects we may use it to distinguish an unsymmetrical face from another having perfect symmetry. A scalene triangle is unsymmetrical, an isosceles triangle is symmetrical but an equilateral triangle is more highly symmetrical than the isosceles triangle. Symmetry is used here as a standard to which observed objects may approximate and which can have different degrees of its own perfect quality.

7. This kind of symmetry implies the possibility of transforming one part of a figure or body into another part by applying to it a prescribed operation, such as mirroring. By mirroring a right hand I can transpose it into a left hand hence a body with two hands is symmetrical. The fact that an equilateral triangle is more symmetrical than an isosceles may be expressed by pointing out that it has three planes of symmetry instead of one. Alternatively, we may introduce a new symmetry operation by observing that the equilateral triangle can be brought to coincide with itself by rotating it by 120° around a vertical axis passing through its centre. We may readily think of symmetry operations for other regular figures and in extension of the same principle, also for regular polyhedra.

The example of the equilateral triangle shows that the presence of three planes of symmetry crossing each other along one line and forming angles of 120° with each other will turn their crossing line into a threefold axis of symmetry. The geometry of regular solids explores such relationships between co-existing elementary symmetries and determines the possibilities for combining such symmetries in one and the same polyhedron. The principles of crystal symmetry was discovered by assuming

that crystals contained only six elementary symmetries (mirroring, inversion and twofold, threefold, fourfold, and sixfold rotations) concluding therefrom that the 32 possible combinations of these six elementary symmetries represented all distinct kinds of crystal symmetry. Operating within this system of symmetries the law of rational indices which I described earlier in this lecture, designates all possible ideal polyhedra corresponding to individual crystal specimens.

The only sharp distinctions laid down by this theory is that between the 32 classes of symmetry. They are distinct forms of a certain kind of order. Just as the ideal polyhedron of a crystal specimen, exhaustively represents the regularity of a crystal specimen, so the class of symmetry into which the polyhedron falls, exhaustively represents the regularity of the polyhedron. Just as the same polyhedron could fit innumerable specimens disfigured by different flaws, the same class of symmetry can be embodied in innumerable polyhedra constituted by an indefinite series of surfaces having an infinite range of relative extension.

Each class of symmetry is a distinctive standard of perfect order to which observed specimens approximate but these standards possess different degrees of their own form of perfection. The 32 classes of symmetry can be arranged roughly in a line of descending symmetries, from the highest cubic to the lowest triclinic class. The variation down this series is extensive and only the higher classes possess sufficient beauty to make their specimens valued as precious stones.

We have here, in brief, the exhaustive formalisation of our appreciation of regularity in crystals, including that of the existence of distinctive kinds of such regularities and of the different grades of regularity represented by each kind. I

shall postpone a further analysis of the relation of this formalism to experience until I have supplemented it by an account of the hidden structural pattern of which it is today regarded as the overt manifestation.

8. The atomic theory of crystals defining this structure which was prophetically mooted in the 19th and triumphantly vindicated early in the 20th century has unified and greatly extended the system of order enframed in the 32 classes of symmetry. In this theory the significance of the planes and edges exhibited by a crystal is further reduced. These distinctive features are now regarded as merely indicating the presence of an underlying atomic orderliness from which the 32 classes of symmetry and the law of rational indices can all be rigorously derived.

The principle of atomic orderliness is an extension of the conception of symmetry. If an operation which brings one part of a figure into coincidence with another part of it is defined as productive of a symmetry, a repetitive pattern like that of a wall paper may be regarded as symmetrical in view of the fact that its parallel displacement brings it to coincide with itself, except for the edges, which we may disregard if the sheet is very large compared with the spacing of the pattern. The elementary symmetry possessed by a regular repetitive pattern may be called translational identity; it can be readily conceived of in one, two three or more dimensions. The structural theory of crystals assumes that they are built as regularly repetitive three dimensional arrays of atoms.

Such arrays, when taken to extend in all directions to infinity, can be readily seen to possess symmetries of the kind observed in crystals, and it can be proved that they can possess only those six elementary symmetries which are found in crystals.

Owing to certain alternative possibilities of regular atomic structure not affecting the symmetry of the crystal as observed macroscopically, the underlying three dimensional atomic patterns can have 230 distinctive rhythms; which are manifested in only 32 distinctive principles of crystalline regularity. The law of rational indices is obtained from the plausible physcial assumption that planes which within a crystal lattice are more densely packed with atoms have a tendendy for appearing as plane boundaries of a crystal constituted by such a lattice.

By Xray analysis we can determine the atomic pattern of a crystelline material without paying any attention to the presence of crystal surfaces, which may be completely absent. The pursuit of this method has gone far towards substantiating the existence of the postulated 230 principles of atomic order.

I am inclined to accept this achievement, and our whole knowledge of crystal structures for which sets the framework, as the supreme vindication of the primitive attraction exercised by crystals and of the intimation entailed in it of a hidden principle underlying their peculiar shapeliness.

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We may now turn more closely to the question, on what principles our acceptance of crystallographic theory rests; while avoiding as far as possible the repetition of what has already been said about empirical conditions for our acceptance of and continued belief in the rule of rational indices.

The theories of the 32 classes of symmetry and of the 230 repetitive patterns called 'space groups' are geometrical statements. As such they speak in terms defined only by the fact that they satisfy the axioms of the theory. The spatial pictures by which we keep in mind their meaning are merely a possible model which embodies this meaning. However, geometry even in the form of a spatial model, says nothing about

experience. Its acceptance rests on our validation of its consistency, ingenuity and profundity. But it does bear potentially on experience, for there is always a possibility that experience may present us with models for a geometrical theory. Such experience may be contrived, consisting in an artificial model or alternatively, the interpretation of a geometry may be found in the natural order of things. Our conceptual imagination like its artistic counterpart, draws inspiration from contacts with experience. And like the works of imaginative art, the constructions of mathematics will tend therefore to disclose those hidden principles of the experienced world of which some scattered traces had first stimulated the poetic process by which they were conceived. Euclidean geometry arose in this sense from experience, when Greek mathematics devised a deductive system that seemed capable of being embodied in the behaviour of observable things insofar as they are, in approximation, ideal solids. We know also how this construction proved deeper than its first understanding by its authors. Like a hen, terrified by seeing its brood of ducklings approaching the water, Euclidean geometry insisted on the necessity of its postulates, while its major scope lay open precisely by casting loose from this very assumption. When at last geometry realised its capacity for modifying its axioms, it released such new powers of conceptual construction that it anticipated the formal features of relativistic dynamics. Among the new geometries that had thus become conceivable there was one which, embodies in terms of experience, proved to represent a system of dynamics far surpassing the hitherto accepted system in coherence and power. When experience is taken to be an embodiment of geometry, geometry comes to be regarded as the image of experience and it may become possible to test its correspondence to experience. The observation of relativistic phenomena has served

as an experimental test for deciding whether the material universe was an instance of Riemann's geometry formulated in space-time by Einstein's rules (when combined with the assumption of trajectories being geodetics.)

The 32 Classes of symmetry and the 230 space groups form a geometric system which can be interpreted in terms of three dimensional relations between geometric elements. The 32 classes define groups of polyhedra and the 230 space groups define indefinitely extended patterns of points in space. These geometrical constructions were originally initiated by a contemplation of crystals and speculations about their atomic structure; hence they will tend to refer to these matters of experience and it is in the following up of this reference by observation that any empirical grounds for our acceptance of crystallographic theory must be found.

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We must deal separately with the 32 classes of symmetry and the 230 space-groups. First as to the former. There are two ways in which experience might invalidate this formalism. On the one hand I can cimagine a solid bounded by plane surfaces and possessing a five fold axis of symmetry. This would represent a crystal which does not fall into any of the 32 classes of symmetry, claimed by crystallographic theory to comprise an exhaustive list of types of crystalline symmetry. (1) The finding of such a crystal would conflict with the theory and might lead to its modification. (2) Suppose on the other hand that all crystals were found to belong to one single class of the 32 alleged to be possible. Though this would not conflict with the system it might fatally weaken it by rendering it practically pointless.

The theory of space-groups could be invalidated by experience only in the latter way. Its claims to be a complete system of all possible atomic lattices might conceivably be

mistaken on geometrical grounds and this may become apparent through the observation of a lattice which is not classifiable in any of the 230 space-groups. But supposing that the geometric deduction is, on its own premises, correct; then experience can only teach us whether or not there are in the world instances of atomic structure which embody these premises. There may exist an infinite range of bodies which do not embody them, among them even some (like disorderly solid solutions) which form externally well shaped crystals; yet this would reveal no internal inconsistency and therefore cause no embarrassment to the theory. Therefore, no conceivable event could falsify this theory. The relation of crystallographic theory to experience is similar in this respect to that between alternative geometries to the actually experienced universe. I have mentioned this parallelism in the previous section. An obvious difference between the two relations of theory and experience lies in the fact that there exists only one single material universe which can serve as an instance of one among many possible geometries, while there exist a great many crystals each of which is an instance of one out of 230 possible space groups, comprising together one unitary theory. The relation of theory to experience is in this respect more akin to that between a classificatory system, such as used by zoologists or botanists and the specimens classified by them. But in view of the fact that the classification is based in the present case on an antecedent geometrical theory of order, the relation between theory and experience appears also akin to that established by a work of art which makes us see experience in its light. I shall now survey these several affiliations and discriminations in order to define the distinctive relation between theory and experience for the case of crystallographic theory.

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The system of 230 space groups was originally developed

in two successive stages, the first by Soncke ( ) and the second by Fedorow and Schonfliess ( ) at a time when there was no method available for observing the arrangement of atoms in crystals and there were no grounds to expect that factual instances of the postulated space groups could ever be identified. The theory of the 230 space groups was accepted by mathematicians mainly on account of its geometrical interest, which was rendered only a little more vivid by the fact that its conceptions could claim to apply to the hypothetical orderly structure of atoms in crystals. Before Laue's discovery of the diffraction of Xrays in crystals there was little speculation on the atomic pattern of crystals, and such as took place was merely conjectural, evoking little response among scientists.

Soon after the Xray analysis of crystals first got under way the theory of space groups was adopted as a guide to systematic crystal analysis. By assuming that the geometrical laws of the space groups governed the arrangement of the atoms in crystals the observed pattern of the Xrays diffracted by a crystal was interpreted in terms of a hypothetical pattern of atoms in the crystals. On this assumption the Xray patterns of different crystals were found to correspond to atomic arrangements representing a variety of space groups, spread widely over the system of 230 theoretical possibilities.

These results did not stand by themselves, but formed the framework of manifold investigations leading to a rich and quantitatively determinate picture of the crystalline state. Yet we may take the process of assigning a crystal to a space group separately and regard it as a process of acquiring a combined knowledge of the crystal and the space-group. The possibility of this knowledge is implied by the theory of space groups and to this extent its acceptance confirms the theory and

shows at the same time that the acceptance of the theory depends to some extent on the facts.

Here stands revealed a system of personal knowledge referring to experience, to which the conception of falsifiability seems altogether inapplicable. Facts which are not described by the theory create no difficulty for the theory for it regards them as irrelevant to itself. Such a theory functions as a comprehensive idiom which consolidates that experience to which it is apposite and leaves unheeded whatever is not comprehended by it.

This process is akin to the taxonomist's performance in identifying a zoological or botanical specimen. But it is a priori and exhaustive. The classification of objects according to a set of preconceived kinds implies an affirmation about matters of experience which may be more or less far-reaching.

A classification is interesting if it tells us a great deal about an object once this is identified as belonging to one of its classes. Such a system is said to classify objects according to their distinctive nature. The distinctiveness of the 230 space groups, like that of the 32 classes of crystal symmetry rests purely on our appreciation of order. They embody in terms of specific symmetries the claim to universality which we necessarily attach to our personal conceptions of order.

This order is not apprehended by our sensuous imagination but is defined conceptually by an apposite formalism. This discriminates it from the realm of art, which it otherwise resembles in several essential respects. We have in both cases an initial summary contact with experience supplying the theme of a subsequent creative performance. The elaborate deduction of the systems of symmetry is validated in the first place by the intellectual pleasure offered by its internal context; the enjoyment of its manifold ingenuity and the fascination of its

deeper implications, as yet only dimly perceived. Its status so far is similar to that of an abstract work of art which merely teaches us to appreciate itself. However, all art makes us also see experience in its own light, and therein lies a further process of its validation. Similarly, the theory of symmetries validates and is validated by a wide area of experience; it controls altogether the collection, description, classification and structural analysis of crystals and is immensely strengthened by this performance. But this application of crystallographic theory to experience is open to the hazards of empirical refutation only in the same sense as a marching song played by the band at the head of a marching column. If it is not found apposite it will not be popular.

Crystallographic theory may in this sense be said to transcend the experience to which it applies. It is not assumed that real crystals ever have the properties of symmetry prescribed in this geometrical theory; any more than engineering assumes that a steam engine can run without friction or economics assume perfect markets to exist in reality. The symmetries in question are idealisations from which various deviations are known to occur which are systematically studied as so many different imperfections. Transcendence which renders an empirical theory irrefutable by experience is of course present in every form of idealisation. The theory of ideal gases cannot be disproved by observed deviations from it, so long as they are of the kind which we are supposed to disregard. Such idealisations do in fact express an element of the same contemplative apprecaition of which the a priori constructions and acceptance of a complete system of symmetries is a fully constituted example. We can be legitimately attracted by the concept of ideal gases only inasmuch as we

believe in our capacity of appreciating a kind of fundamental orderliness in nature which underlies some of its less orderly appearances. But in the theory of crystal symmetries idealisation goes beyond this. For the standards of excellence which are developed by this system possess a much higher degree of instrinsic significance than the formula pv= RT may claim for itself. It is not merely a scientific idealisation but the formalisation of an esthetic ideal, closely akin to that deeper and never rigidly definable sensibility by which the domains of art and art-criticism are governed. That is why this theory teaches us to appreciate certain things, regardless of whether we may find any of their kind in nature; as well as allows us to criticise these things when we find them to the extent to which they fall short of the standards which the theory sets to nature.

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Such discourse is not about experience, but legislates for experience, as it sets out the criteria by which experience shall be appreciated. It is an exposition of the system of ideals which we accept for the pursuit of empirical crystallography. Within this pursuit we are committed to this system of idealisation and the exposition of the latter stands for a confirmation of this commitment. However, the acceptance of such a system may be, as in this instance it once was, purely theoretical, if its internal context impresses us by its ingenuity and the profundity of its yet unexplored implications. We may then pursue the study of our idealisations systematically for its own sake, without closer interest in their application to experience as criteria of its perfection. Thus a four-dimensional crystallography might be deemed interesting on purely theoretical grounds, without any plausible hope of finding an application for it.

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13. We have staked out within the exact sciences fields in which personal knowledge takes an increasingly prominent part. Looking back over this survey, in conclusion, we may mark our acknowledgement of personal participation throughout these disciplines by amending the conventional distinction between theory and fact in favour of a distinction between personal fact and objective fact. When I speak of a fact I always mean something I believe to be a fact. By describing it as a fact I express my conviction that it objectively exists. The facts of classical dynamics are measured quantities referred to by a theoretical formalism. They are accepted as facts in the light of this theory, but the theory itself is not a fact of the same kind. It is not ascertained by measurement but accepted as a result of our appreciation of its rationality. We might say that it is not an objective fact but a personal fact. Things to which our personal knowledge refers such as the numerical stated probability of an event, simple integer proportions observed by measurement and the symmetry of crystals and atomic patterns are then all classed as personal facts. Although the difference between objective and personal facts lies only in the degree of our personal participation, this participation is so much more marked in what I have called personal facts that the distinction may justly be taken as one of kind.

14. This concludes my survey of the exact sciences. I shall proceed to consider the functioning of personal knowledge in manifestations of human intelligence which lie beyond the sciences, in order finally to look at the performances of living things in general, and to lay a foundation for assessing, in this context, the responsible actions of men, of which both science and philosophy, as exemplified on the one hand by the formal disciplines I have been examining, and on the other by my reflective appraisal of them, constitute but a narrow segment.